

STRUCTURE OF THE $(0^+, 1^+)$ MESONS B_{s0} AND B_{s1} , AND THE STRONG COUPLING CONSTANTS $g_{B_{s0}BK}$ AND $g_{B_{s1}B^*K}$

Z. G. Wang¹

Department of Physics, North China Electric Power University, Baoding 071003,
P. R. China

Abstract

In this article, we take the point of view that the bottomed $(0^+, 1^+)$ mesons B_{s0} and B_{s1} are the conventional $b\bar{s}$ meson, and calculate the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ with the light-cone QCD sum rules. The numerical values of strong coupling constants $g_{B_{s1}B^*K}$ and $g_{B_{s0}BK}$ are very large, and support the hadronic dressing mechanism. Just like the scalar mesons $f_0(980)$, $a_0(980)$, D_{s0} and axial-vector meson D_{s1} , the $(0^+, 1^+)$ bottomed mesons B_{s0} and B_{s1} may have small $b\bar{s}$ kernels of the typical $b\bar{s}$ meson size, the strong couplings to the hadronic channels (or the virtual mesons loops) may result in smaller masses than the conventional $b\bar{s}$ mesons in the potential quark models, and enrich the pure $b\bar{s}$ states with other components.

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1 Introduction

Recently, the CDF Collaboration reports the first observation of two narrow resonances consistent with the orbitally excited P -wave B_s mesons using 1 fb^{-1} of $p\bar{p}$ collisions at $\sqrt{s} = 1.96 \text{ TeV}$ collected with the CDF II detector at the Fermilab Tevatron [1]. The masses of the two states are $M(B_{s1}) = (5829.4 \pm 0.7) \text{ MeV}$ and $M(B_{s2}^*) = (5839.7 \pm 0.7) \text{ MeV}$, and they can be assigned as the $J^P = (1^+, 2^+)$ states in the heavy quark effective theory [2]. The D0 Collaboration reports the direct observation of the excited P -wave state B_{s2}^* in fully reconstructed decays to B^+K^- , the mass of the B_{s2}^* meson is measured to be $(5839.6 \pm 1.1 \pm 0.7) \text{ MeV}$ [3]. While the B_s states with spin-parity $J^P = (0^+, 1^+)$ are still lack experimental evidence.

The masses of the B_s mesons with $(0^+, 1^+)$ have been estimated with the potential quark models, heavy quark effective theory and lattice QCD [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], the values are different from each other. In our previous work [17], we study the masses of the bottomed $(0^+, 1^+)$ mesons with the QCD sum rules, and observe that the central values are below the corresponding BK and B^*K thresholds respectively. The strong decays $B_{s0} \rightarrow BK$ and $B_{s1} \rightarrow B^*K$ are kinematically forbidden, the P -wave heavy mesons B_{s0} and B_{s1} can decay through the isospin violation processes $B_{s0} \rightarrow B_s\eta \rightarrow B_s\pi^0$ and $B_{s1} \rightarrow B_s^*\eta \rightarrow B_s^*\pi^0$, respectively [18]. The η and π^0 transition matrix is very small according to Dashen's theorem

¹E-mail, wangzgyiti@yahoo.com.cn.

[19], $t_{\eta\pi} = \langle \pi^0 | \mathcal{H} | \eta \rangle = -0.003 \text{GeV}$, they maybe very narrow. The bottomed mesons B_{s0} and B_{s1} may have interesting feature, just like their charmed cousins D_{s0} and D_{s1} , have small $b\bar{s}$ kernels of the typical $b\bar{s}$ mesons size, strong couplings to the virtual intermediate hadronic states (or the virtual mesons loops) may result in smaller masses than the conventional $b\bar{s}$ mesons in the potential quark models, enrich the pure $b\bar{s}$ states with other components [14, 20, 21, 22, 23, 24].

In previous works, the mesons $f_0(980)$, $a_0(980)$, D_{s0} and D_{s1} are taken as the conventional $q\bar{q}$ and $c\bar{s}$ states respectively, and the values of the strong coupling constants g_{f_0KK} , g_{a_0KK} , $g_{D_{s0}DK}$ and $g_{D_{s1}D^*K}$ are calculated with the light-cone QCD sum rules [22, 23, 24, 25, 26]. The large values of the strong coupling constants support the hadronic dressing mechanism.

In this article, we take the bottomed mesons B_{s0} and B_{s1} as the conventional $b\bar{s}$ states, and calculate the values of the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ with the light-cone QCD sum rules, and study the possibility of the hadronic dressing mechanism in the bottomed channels.

The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$ while the non-perturbative matrix elements are parameterized by the light-cone distribution amplitudes (which classified according to their twists) instead of the vacuum condensates [27, 28, 29, 30, 31, 32]. The non-perturbative parameters in the light-cone distribution amplitudes are calculated by the conventional QCD sum rules and the values are universal [33, 34, 35, 36].

The article is arranged as: in Section 2, we derive the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and in Section 4, conclusion.

2 Strong coupling constants $g_{B_{s1}B^*K}$ and $g_{B_{s0}BK}$ with light-cone QCD sum rules

In the following, we write down the definition for the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$,

$$\begin{aligned}\langle B_{s1} | B^* K \rangle &= -ig_{B_{s1}B^*K} \eta^* \cdot \epsilon = -iM_A \hat{g}_{B_{s1}B^*K} \eta^* \cdot \epsilon , \\ \langle B_{s0} | BK \rangle &= g_{B_{s0}BK} = M_S \hat{g}_{B_{s0}BK} ,\end{aligned}\tag{1}$$

where the ϵ_α and η_α are the polarization vectors of the mesons B^* and B_{s1} respectively. The masses M_S and M_A can serve as an energy scale, we factorize the masses from the corresponding strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ respectively.

We study the strong coupling constants with the two-point correlation functions

$\Pi_{\mu\nu}(p, q)$ and $\Pi_\mu(p, q)$ respectively,

$$\Pi_{\mu\nu}(p, q) = i \int d^4x e^{-iq\cdot x} \langle 0 | T \{ J_\mu^V(0) J_\nu^{A+}(x) \} | K(p) \rangle, \quad (2)$$

$$\Pi_\mu(p, q) = i \int d^4x e^{-iq\cdot x} \langle 0 | T \{ J_\mu^5(0) J_\nu^{S+}(x) \} | K(p) \rangle, \quad (3)$$

$$\begin{aligned} J_\mu^V(x) &= \bar{u}(x) \gamma_\mu b(x), \\ J_\mu^A(x) &= \bar{s}(x) \gamma_\mu \gamma_5 b(x), \\ J_\mu^5(x) &= \bar{u}(x) \gamma_\mu \gamma_5 b(x), \\ J^S(x) &= \bar{s}(x) b(x), \end{aligned} \quad (4)$$

where the currents $J_\mu^V(x)$, $J_\mu^A(x)$, $J_\mu^5(x)$ and $J^S(x)$ interpolate the bottomed mesons B^* , B_{s1} , B and B_{s0} , respectively, the external K meson has four momentum p_μ with $p^2 = m_K^2$. The correlation functions $\Pi_{\mu\nu}(p, q)$ and $\Pi_\mu(p, q)$ can be decomposed as

$$\begin{aligned} \Pi_{\mu\nu}(p, q) &= i\Pi_A(p, q)g_{\mu\nu} + \Pi_{A1}(p, q)(p_\mu q_\nu + p_\nu q_\mu) + \dots, \\ \Pi_\mu(p, q) &= i\Pi_S(p, q)q_\mu + \Pi_{S1}(p, q)p_\mu + \dots \end{aligned} \quad (5)$$

due to the Lorentz invariance.

According to the basic assumption of current-hadron duality in the QCD sum rules approach [33, 34, 35, 36], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_\mu^V(x)$, $J_\mu^A(x)$, $J_\mu^5(x)$ and $J^S(x)$ into the correlation functions $\Pi_{\mu\nu}(p, q)$ and $\Pi_\mu(p, q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the mesons B^* , B_{s1} , B and B_{s0} , we get the following results,

$$\begin{aligned} \Pi_{\mu\nu} &= \frac{\langle 0 | J_\mu^V(0) | B^*(q+p) \rangle \langle B^* | B_{s1} K \rangle \langle B_{s1}(q) | J_\nu^{A+}(0) | 0 \rangle}{[M_{B^*}^2 - (q+p)^2] [M_A^2 - q^2]} + \dots \\ &= -\frac{i g_{B_{s1} B^* K} f_{B^*} f_A M_{B^*} M_A}{[M_{B^*}^2 - (q+p)^2] [M_A^2 - q^2]} g_{\mu\nu} + \dots, \end{aligned} \quad (6)$$

$$\begin{aligned} \Pi_\mu &= \frac{\langle 0 | J_\mu^5(0) | B(q+p) \rangle \langle B | B_{s0} K \rangle \langle B_{s0}(q) | J^{S+}(0) | 0 \rangle}{[M_B^2 - (q+p)^2] [M_S^2 - q^2]} + \dots \\ &= \frac{i g_{B_{s0} B K} f_B f_S M_S}{[M_B^2 - (q+p)^2] [M_S^2 - q^2]} (p+q)_\mu + \dots, \end{aligned} \quad (7)$$

where the following definitions for the weak decay constants have been used,

$$\begin{aligned} \langle 0 | J_\mu^V(0) | B^*(p) \rangle &= f_{B^*} M_{B^*} \epsilon_\mu, \\ \langle 0 | J_\mu^A(0) | B_{s1}(p) \rangle &= f_A M_A \eta_\mu, \\ \langle 0 | J_\mu^5(0) | B(p) \rangle &= i f_B p_\mu, \\ \langle 0 | J^S(0) | B_{s0}(p) \rangle &= f_S M_S. \end{aligned} \quad (8)$$

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi_{\mu\nu}(p, q)$ and $\Pi_\mu(p, q)$ in perturbative QCD theory. The calculations are performed at the large space-like momentum regions $(q+p)^2 \ll 0$ and $q^2 \ll 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the operator product expansion approach. We write down the propagator of a massive quark in the external gluon field in the Fock-Schwinger gauge firstly [37],

$$\begin{aligned} \langle 0 | T\{q_i(x_1) \bar{q}_j(x_2)\} | 0 \rangle &= i \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x_1-x_2)} \\ &\left\{ \frac{\not{k} + m}{k^2 - m^2} \delta_{ij} - \int_0^1 dv g_s G_{ij}^{\mu\nu}(vx_1 + (1-v)x_2) \right. \\ &\left. \left[\frac{1}{2} \frac{\not{k} + m}{(k^2 - m^2)^2} \sigma_{\mu\nu} - \frac{1}{k^2 - m^2} v(x_1 - x_2)_\mu \gamma_\nu \right] \right\}. \quad (9) \end{aligned}$$

Substituting the above b quark propagator and the corresponding K meson light-cone distribution amplitudes into the correlation functions $\Pi_{\mu\nu}(p, q)$ and $\Pi_\mu(p, q)$, and completing the integrals over the variables x and k , finally we obtain the analytical results, which are given explicitly in the appendix.

In calculation, the two-particle and three-particle K meson light-cone distribution amplitudes have been used [38, 39, 40, 41], the explicit expressions are given in the appendix. The parameters in the light-cone distribution amplitudes are scale dependent and are estimated with the QCD sum rules [38, 39, 40, 41]. In this article, the energy scale μ is chosen to be $\mu = 1\text{GeV}$, to be more precise, one can choose $\mu = \sqrt{M_B^2 - m_b^2} \approx 2.4\text{GeV}$.

After straightforward calculations, we obtain the final expressions of the double Borel transformed correlation functions $\Pi_A(M_1^2, M_2^2)$ and $\Pi_S(M_1^2, M_2^2)$ at the level of quark-gluon degrees of freedom. The masses of the bottomed mesons are $M_A = 5.72\text{GeV}$, $M_S = 5.70\text{GeV}$, $M_{B^*} = 5.33\text{GeV}$ and $M_B = 5.28\text{GeV}$,

$$\frac{M_A^2}{M_A^2 + M_{B^*}^2} \approx \frac{M_S^2}{M_S^2 + M_B^2} \approx 0.54, \quad (10)$$

there exists an overlapping working window for the two Borel parameters M_1^2 and M_2^2 , it's convenient to take the value $M_1^2 = M_2^2$. We introduce the threshold parameter s_0 and make the simple replacement,

$$e^{-\frac{m_b^2 + u_0(1-u_0)m_K^2}{M^2}} \rightarrow e^{-\frac{m_b^2 + u_0(1-u_0)m_K^2}{M^2}} - e^{-\frac{s_0}{M^2}}$$

to subtract the contributions from the high resonances and continuum states [37], finally we obtain the sum rules for the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$,

$$\begin{aligned}
g_{B_{s0}BK} &= \frac{1}{f_B f_S M_S} \exp \left(\frac{M_S^2}{M_1^2} + \frac{M_B^2}{M_2^2} \right) \left\{ \left[\exp \left(-\frac{\Xi}{M^2} \right) - \exp \left(-\frac{s_S^0}{M^2} \right) \right] \right. \\
&\quad \frac{f_K m_K^2 M^2}{m_u + m_s} \left[\varphi_p(u_0) - \frac{d\varphi_\sigma(u_0)}{6du_0} \right] + \exp \left(-\frac{\Xi}{M^2} \right) \left[-m_b f_K m_K^2 \int_0^{u_0} dt B(t) \right. \\
&\quad \left. + f_{3K} m_K^2 \int_0^{u_0} d\alpha_s \int_{u_0-\alpha_s}^{1-\alpha_s} d\alpha_g \varphi_{3K}(1-\alpha_s-\alpha_g, \alpha_g, \alpha_s) \frac{2(\alpha_s + \alpha_g - u_0) - 3\alpha_g}{\alpha_g^2} \right. \\
&\quad \left. - \frac{2m_b f_K m_K^4}{M^2} \int_{1-u_0}^1 d\alpha_g \frac{1-u_0}{\alpha_g^2} \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \Phi(1-\alpha-\beta, \beta, \alpha) \right. \\
&\quad \left. + \frac{2m_b f_K m_K^4}{M^2} \left(\int_0^{1-u_0} d\alpha_g \int_{u_0-\alpha_g}^{u_0} d\alpha_s \int_0^{\alpha_s} d\alpha + \int_{1-u_0}^1 d\alpha_g \int_{u_0-\alpha_g}^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \right) \right. \\
&\quad \left. \left. \frac{\Phi(1-\alpha-\alpha_g, \alpha_g, \alpha)}{\alpha_g} \right) \right\}, \tag{11}
\end{aligned}$$

$$\begin{aligned}
g_{B_{s1}B^*K} &= \frac{1}{f_{B^*}f_AM_{B^*}M_A} \exp\left(\frac{M_A^2}{M_1^2} + \frac{M_{B^*}^2}{M_2^2}\right) \left\{ \left[\exp\left(-\frac{\Xi}{M^2}\right) - \exp\left(-\frac{s_A^0}{M^2}\right) \right] \right. \\
&\quad f_K \left[\frac{m_b m_K^2 M^2}{m_u + m_s} \varphi_p(u_0) + \frac{m_K^2(M^2 + m_b^2)}{8} \frac{d}{du_0} A(u_0) - \frac{M^4}{2} \frac{d}{du_0} \phi_K(u_0) \right] \\
&\quad - \exp\left(-\frac{\Xi}{M^2}\right) \left[f_K m_b^2 m_K^2 \int_0^{u_0} dt B(t) \right. \\
&\quad + m_K^2 \int_0^{u_0} d\alpha_s \int_{u_0-\alpha_s}^{1-\alpha_s} d\alpha_g \frac{(u_0 f_K m_K^2 \Phi + f_{3K} m_b \varphi_{3K})(1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g)}{\alpha_g} \\
&\quad + f_K m_K^2 M^2 \frac{d}{du_0} \int_0^{u_0} d\alpha_s \int_{u_0-\alpha_s}^{1-\alpha_s} d\alpha_g \frac{(A_{||} - V_{||})(1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g)}{2\alpha_g} \\
&\quad - f_K m_K^2 M^2 \frac{d}{du_0} \int_0^{u_0} d\alpha_s \int_{u_0-\alpha_s}^{1-\alpha_s} d\alpha_g A_{||}(1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g) \frac{\alpha_s + \alpha_g - u_0}{\alpha_g^2} \\
&\quad + f_K m_K^4 \left(\int_0^{1-u_0} d\alpha_g \int_{u_0-\alpha_g}^{u_0} d\alpha_s \int_0^{\alpha_s} d\alpha + \int_{1-u_0}^1 d\alpha_g \int_{u_0-\alpha_g}^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \right) \\
&\quad \left. \left[\frac{1}{\alpha_g} \left(3 - \frac{2m_b^2}{M^2} \right) \Phi + \frac{4m_b^2}{M^2} \frac{\alpha_s + \alpha_g - u_0}{\alpha_g^2} (A_{\perp} + A_{||}) \right] (1 - \alpha - \alpha_g, \alpha, \alpha_g) \right. \\
&\quad - f_K m_K^4 u_0 \frac{d}{du_0} \left(\int_0^{1-u_0} d\alpha_g \int_{u_0-\alpha_g}^{u_0} d\alpha_s \int_0^{\alpha_s} d\alpha + \int_{1-u_0}^1 d\alpha_g \int_{u_0-\alpha_g}^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \right) \\
&\quad \left. \frac{\Phi(1 - \alpha - \alpha_g, \alpha, \alpha_g)}{\alpha_g} \right. \\
&\quad - f_K m_K^4 \int_{1-u_0}^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \left[\Phi(1 - \alpha - \beta, \alpha, \beta) \frac{1 - u_0}{\alpha_g^2} \left(4 - \frac{2m_b^2}{M^2} \right) \right. \\
&\quad \left. + \frac{4m_b^2}{M^2} \frac{(1 - u_0)^2}{\alpha_g^3} (A_{||} + A_{\perp})(1 - \alpha - \beta, \alpha, \beta) \right] \\
&\quad \left. + f_K m_K^4 \frac{d}{du_0} \int_{1-u_0}^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \Phi(1 - \alpha - \beta, \alpha, \beta) \frac{u_0(1 - u_0)}{\alpha_g^2} \right\}, \quad (12)
\end{aligned}$$

where

$$\begin{aligned}
\Xi &= m_b^2 + u_0(1 - u_0)m_K^2, \\
u_0 &= \frac{M_1^2}{M_1^2 + M_2^2}, \\
M^2 &= \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}.
\end{aligned} \quad (13)$$

The term proportional to the $M^4 \frac{d}{du_0} \phi_K(u_0)$ in Eq.(12) depends heavily on the asymmetric coefficient $a_1(\mu)$ of the twist-2 light-cone distribution amplitude $\phi_K(u)$ in the

limit $u_0 = \frac{1}{2}$ (see also the sum rules for the strong coupling constant $g_{D_{s1}D^*K}$ in Ref. [23]), if we take the value $a_1(\mu) = 0.06 \pm 0.03$ [38, 39, 40, 41], no stable sum rules can be obtained, the value of the $g_{B_{s1}B^*K}$ changes significantly with the variation of the Borel parameter M^2 . In this article, we take the assumption that the u and s quarks have symmetric momentum distributions and neglect the coefficient $a_1(\mu)$.

In the heavy quark limit $m_b \rightarrow \infty$,

$$\begin{aligned}
s_S^0 &\rightarrow m_b^2 + 2m_b\omega_S^0, \\
s_A^0 &\rightarrow m_b^2 + 2m_b\omega_A^0, \\
M_1^2 &\rightarrow 2m_bT_1, \\
M_2^2 &\rightarrow 2m_bT_2, \\
M^2 &\rightarrow 2m_bT, \\
M_S &\rightarrow m_b + \Lambda_1, \\
M_A &\rightarrow m_b + \Lambda_1, \\
M_B &\rightarrow m_b + \Lambda_0, \\
M_{B^*} &\rightarrow m_b + \Lambda_0,
\end{aligned} \tag{14}$$

the two sum rules in Eqs.(11-12) are reduced to the following form,

$$\begin{aligned}
g_{B_{s0}BK} &= \frac{1}{f_B f_S} \exp\left(\frac{\Lambda_1}{T_1} + \frac{\Lambda_0}{T_2}\right) \left\{ \left[1 - \exp\left(-\frac{\omega_S^0}{T}\right) \right] \right. \\
&\quad \left. \frac{2f_K m_K^2 T}{m_u + m_s} \left[\varphi_p(u_0) - \frac{d\varphi_\sigma(u_0)}{6du_0} \right] - f_K m_K^2 \int_0^{u_0} dt B(t) \right\},
\end{aligned} \tag{15}$$

$$\begin{aligned}
g_{B_{s1}B^*K} &= \frac{1}{f_{B^*} f_A} \exp\left(\frac{\Lambda_1}{T_1} + \frac{\Lambda_0}{T_2}\right) \left\{ \left[1 - \exp\left(-\frac{\omega_A^0}{T}\right) \right] \right. \\
&\quad f_K \left[\frac{2m_K^2 T}{m_u + m_s} \varphi_p(u_0) + \frac{m_K^2 (2T + m_b)}{8m_b} \frac{d}{du_0} A(u_0) - 2T^2 \frac{d}{du_0} \phi_K(u_0) \right] \\
&\quad \left. - f_K m_K^2 \int_0^{u_0} dt B(t) \right\},
\end{aligned} \tag{16}$$

where the decay constants take the behavior $f_A = \frac{C_1}{\sqrt{M_A}}$, $f_S = \frac{C_1}{\sqrt{M_S}}$, $f_B = \frac{C_2}{\sqrt{M_B}}$ and $f_{B^*} = \frac{C_2}{\sqrt{M_{B^*}}}$ (according to the definition in Eq.(8)), the C_i are some constants.

3 Numerical result and discussion

The input parameters are taken as $m_s = (140 \pm 10)\text{MeV}$, $m_u = (5.6 \pm 1.6)\text{MeV}$, $m_b = (4.7 \pm 0.1)\text{GeV}$, $\lambda_3 = 1.6 \pm 0.4$, $f_{3K} = (0.45 \pm 0.15) \times 10^{-2}\text{GeV}^2$, $\omega_3 = -1.2 \pm 0.7$, $\eta_4 = 0.6 \pm 0.2$, $\omega_4 = 0.2 \pm 0.1$, $a_2 = 0.25 \pm 0.15$ [32, 38, 39, 40, 41], $f_K = 0.160\text{GeV}$, $m_K = 0.498\text{GeV}$, $M_B = 5.279\text{GeV}$, $M_{B^*} = 5.325\text{GeV}$ [42], $M_S = (5.70 \pm 0.11)\text{GeV}$, $M_A = (5.72 \pm 0.09)\text{GeV}$, $f_S = f_A = (0.24 \pm 0.02)\text{GeV}$ [17],

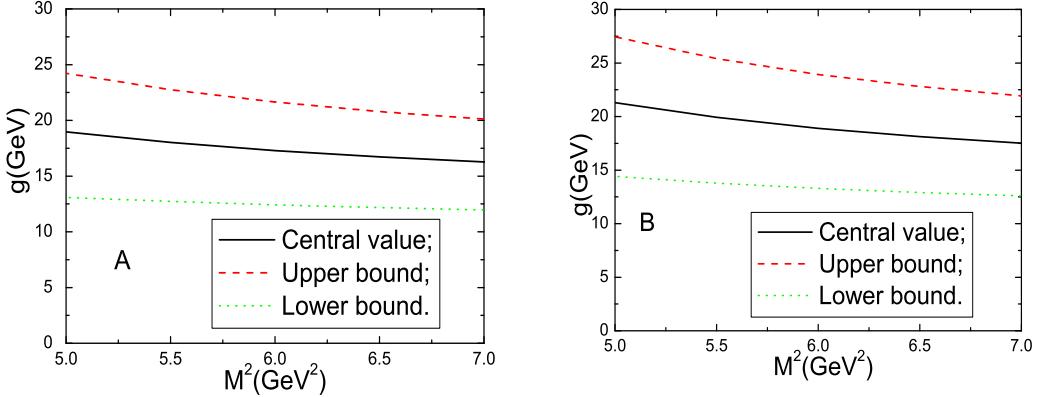


Figure 1: The strong coupling constants $g_{B_{s1}B^{*}K}$ (A) and $g_{B_{s0}BK}$ (B) with the Borel parameter M^2 .

$f_{B^*} = f_B = (0.17 \pm 0.02)\text{GeV}$ [32, 43, 44], $s_S^0 = (37 \pm 1)\text{GeV}^2$, $s_A^0 = (38 \pm 1)\text{GeV}^2$ [17], $\Lambda_0 = \frac{M_B + 3M_{B^*}}{4} - m_b = (0.6 \pm 0.1)\text{GeV}$, $\Lambda_1 = \frac{M_S + 3M_A}{4} - m_b = (1.0 \pm 0.1)\text{GeV}$, $\omega_S^0 = (1.6 \pm 0.1)\text{GeV}$ and $\omega_A^0 = (1.6 \pm 0.1)\text{GeV}$. The Borel parameters are chosen as $M^2 = (5 - 7)\text{GeV}^2$, in this region, the values of the strong coupling constants $g_{B_{s1}B^{*}K}$ and $g_{B_{s0}BK}$ are rather stable, which are shown in Fig.1. In the heavy quark limit, the Borel parameters are chosen as $T = (0.7 - 1.5)\text{GeV}$, in this region, the values of the strong coupling constants $g_{B_{s1}B^{*}K}$ and $g_{B_{s0}BK}$ are rather stable, which are shown in Fig.2.

In the limit of large Borel parameter M^2 , the strong coupling constants $g_{B_{s1}B^{*}K}$ and $g_{B_{s0}BK}$ take up the following behaviors,

$$\begin{aligned} g_{B_{s0}BK} &\propto \frac{M^2 \varphi_p(u_0)}{f_B f_S}, \\ g_{B_{s1}B^{*}K} &\propto \frac{m_b M^2 \varphi_p(u_0)}{f_{B^*} f_A}. \end{aligned} \quad (17)$$

It is not unexpected, the contributions from the two-particle twist-3 light-cone distribution amplitude $\varphi_p(u)$ are greatly enhanced by the large Borel parameter M^2 , (large) uncertainties of the relevant parameters presented in above equations have significant impact on the numerical results. The contributions from the two-particle twist-2, twist-3 and twist-4 light-cone distribution amplitudes $\phi_K(u_0)$, $\varphi_\sigma(u_0)$ and $A(u_0)$ are zero due to symmetry property.

Taking into account all the uncertainties of the input parameters, finally we

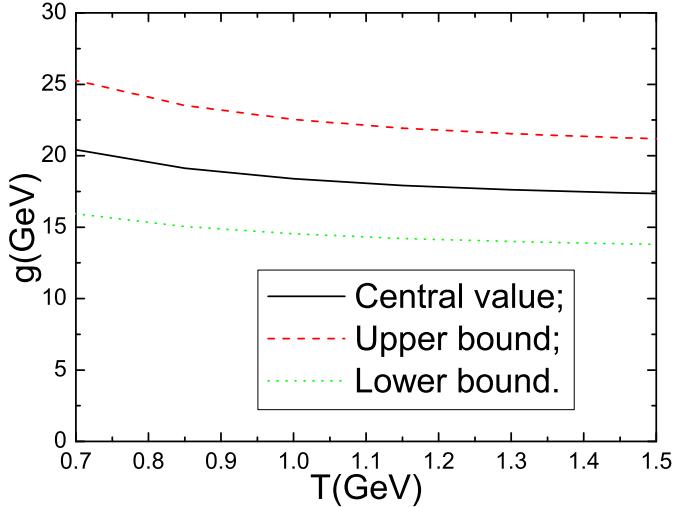


Figure 2: The strong coupling constants $g_{B_{s1}B^*K}$ and $g_{B_{s0}BK}$ with the Borel parameter T in the heavy quark limit.

obtain the numerical values of the strong coupling constants

$$\begin{aligned}
 g_{B_{s1}B^*K} &= (18.1 \pm 6.1)\text{GeV}, \\
 g_{B_{s0}BK} &= (20.0 \pm 7.4)\text{GeV}, \\
 \hat{g}_{B_{s1}B^*K} &= 3.2 \pm 1.1, \\
 \hat{g}_{B_{s0}BK} &= 3.5 \pm 1.3
 \end{aligned} \tag{18}$$

from Eqs.(11-12) and

$$g_{B_{s1}B^*K} = g_{B_{s0}BK} = (19.6 \pm 5.7)\text{GeV} \tag{19}$$

from Eqs.(15-16). The uncertainties are large, about 30%. The contributions from three-particle light-cone distribution amplitudes vanish in the heavy quark limit, the uncertainties are reduced slightly, as the dominating contributions come from the two-particle twist-3 light-cone distribution amplitude $\varphi_p(u)$.

The large values of the strong coupling constants $g_{B_{s1}B^*K}$ and $g_{B_{s0}BK}$ obviously support the hadronic dressing mechanism [45, 46, 47], the scalar meson $B_{s0}(D_{s0})$ and axial-vector meson $B_{s1}(D_{s1})$ can be taken as having small scalar and axial-vector $b\bar{s}$ ($c\bar{s}$) kernels of typical meson size with large virtual S -wave $BK(DK)$ and $B^*K(D^*K)$ cloud respectively.

In Refs.[48, 49], the authors analyze the unitarized two-meson scattering amplitudes from the heavy-light chiral lagrangian, and observe that the scalar mesons D_{s0} and B_{s0} , and axial-vector mesons D_{s1} and B_{s1} appear as the bound state poles with the strong coupling constants $g_{D_{s0}DK} = 10.203\text{GeV}$, $g_{D_{s1}D^*K} = 10.762\text{GeV}$,

	$g_{B_{s1}B^*K}$ (GeV)	$g_{B_{s0}BK}$ (GeV)	$g_{D_{s1}D^*K}$ (GeV)	$g_{D_{s0}DK}$ (GeV)
[22, 23]			10.5 ± 3.5	$9.3^{+2.7}_{-2.1}$
[48, 49]	23.572	23.442	10.762	10.203
This work	18.1 ± 6.1	20.0 ± 7.4		
This work*	19.6 ± 5.7	19.6 ± 5.7		

Table 1: Theoretical estimations of the strong coupling constants from different models, where * stands for the strong coupling constants in the heavy quark limit.

$g_{B_{s1}B^*K} = 23.572\text{GeV}$ and $g_{B_{s0}BK} = 23.442\text{GeV}$. Our numerical results for the strong coupling constants are certainly reasonable and can make robust predictions. However, we take the point of view that the mesons D_{s0} , B_{s0} , D_{s1} and B_{s1} be bound states in the sense that they appear below the corresponding DK , BK , D^*K and B^*K thresholds respectively, their constituents may be the bare $c\bar{s}$ and $b\bar{s}$ states, the virtual DK , BK , D^*K and B^*K pairs and their mixing, rather than the DK , BK , D^*K and B^*K bound states.

In Ref.[50], the authors calculate the strong coupling constants $g_{D_{s0}D_s\eta}$ and $g_{D_{s1}D_s^*\eta}$ with the light-cone QCD sum rules, then take into account $\eta - \pi^0$ mixing and calculate their pionic decay widths. The bottomed mesons B_{s0} and B_{s1} can decay through the same isospin violation mechanism, $B_{s0} \rightarrow B_s\eta \rightarrow B_s\pi^0$ and $B_{s1} \rightarrow B_s^*\eta \rightarrow B_s^*\pi^0$. We study the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$ with the light-cone QCD sum rules and make predictions for the corresponding small decay widths [18].

4 Conclusion

In this article, we take the point of view that the bottomed mesons B_{s0} and B_{s1} are the conventional $b\bar{s}$ mesons and calculate the strong coupling constants $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ with the light-cone QCD sum rules. The numerical results are compatible with the existing estimations, the large values support the hadronic dressing mechanism. Just like the scalar mesons $f_0(980)$, $a_0(980)$, D_{s0} and axial-vector meson D_{s1} , the bottomed mesons B_{s0} and B_{s1} may have small $b\bar{s}$ kernels of typical $b\bar{s}$ meson size. The strong couplings to virtual intermediate hadronic states (or the virtual mesons loops) can result in smaller masses than the conventional 0^+ and 1^+ mesons in the potential quark models, enrich the pure $b\bar{s}$ states with other components.

Appendix

The analytical expressions of the $\Pi_S(p, q)$ and $\Pi_A(p, q)$ at the level of the quark-gluon degrees of freedom,

$$\begin{aligned}
\Pi_S &= \frac{f_K m_K^2}{m_u + m_s} \int_0^1 du \frac{\varphi_p(u)}{\Delta} - m_b f_K m_K^2 \int_0^1 du \int_0^u dt \frac{B(t)}{\Delta^2} \\
&\quad + \frac{1}{6} \frac{f_K m_K^2}{m_u + m_s} \int_0^1 du \varphi_\sigma(u) \frac{d}{du} \frac{1}{\Delta} \\
&\quad + f_{3K} m_K^2 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \varphi_{3K}(\alpha_u, \alpha_g, \alpha_s) \frac{2v-3}{\Delta^2} |_{u=(1-v)\alpha_g+\alpha_s} \\
&\quad - 4m_b f_K m_K^4 \int_0^1 dv v \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \frac{\Phi(1-\alpha-\beta, \beta, \alpha)}{\Delta^3} |_{u=1-v\alpha_g} \\
&\quad + 4m_b f_K m_K^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \frac{\Phi(1-\alpha-\alpha_g, \alpha_g, \alpha)}{\Delta^3} |_{u=(1-v)\alpha_g+\alpha_s}, \\
\Pi_A &= -\frac{f_K m_b m_K^2}{m_u + m_s} \int_0^1 du \frac{\varphi_p(u)}{\Delta} + f_K m_b^2 m_K^2 \int_0^1 du \int_0^u dt \frac{B(t)}{\Delta^2} \\
&\quad + \frac{f_K}{2} \int_0^1 du \left\{ \phi_K(u) \frac{d}{du} \log \Delta + \frac{A(u)m_K^2}{4} \frac{d}{du} \left[\frac{1}{\Delta} + \frac{m_b^2}{\Delta^2} \right] \right\} \\
&\quad + m_K^2 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \\
&\quad \frac{[f_K m_K^2 u \Phi + f_{3K} m_b \varphi_{3K}] (1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g)}{\Delta^2} |_{u=\alpha_s+(1-v)\alpha_g} \\
&\quad - \frac{f_K m_K^2}{2} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s [(1-2v) A_{\parallel} - V_{\parallel}] (1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g) \\
&\quad \frac{d}{du} \frac{1}{\Delta} |_{u=\alpha_s+(1-v)\alpha_g} \\
&\quad + f_K m_K^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \Phi(1-\alpha-\alpha_g, \alpha, \alpha_g) \\
&\quad \left\{ \frac{4}{\Delta^2} - \frac{4m_b^2}{\Delta^3} + u \frac{d}{du} \frac{1}{\Delta^2} \right\} |_{u=\alpha_s+(1-v)\alpha_g} \\
&\quad - f_K m_K^4 \int_0^1 dv v \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \Phi(1-\alpha-\beta, \alpha, \beta) \\
&\quad \left\{ \frac{4}{\Delta^2} - \frac{4m_b^2}{\Delta^3} + u \frac{d}{du} \frac{1}{\Delta^2} \right\} |_{u=1-v\alpha_g} \\
&\quad + 8f_K m_b^2 m_K^4 \int_0^1 dv v \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha (A_{\parallel} + A_{\perp}) (1 - \alpha - \alpha_g, \alpha, \alpha_g) \\
&\quad \frac{1}{\Delta^3} |_{u=\alpha_s+(1-v)\alpha_g} \\
&\quad - 8f_K m_b^2 m_K^4 \int_0^1 dv v^2 \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha (A_{\parallel} + A_{\perp}) (1 - \alpha - \beta, \alpha, \beta) \\
&\quad \frac{1}{\Delta^3} |_{u=1-v\alpha_g}, \tag{20}
\end{aligned}$$

where

$$\begin{aligned}\Delta &= m_b^2 - (q + up)^2, \\ \Phi &= A_{\parallel} + A_{\perp} - V_{\parallel} - V_{\perp}.\end{aligned}\quad (21)$$

The light-cone distribution amplitudes of the K meson are defined by

$$\begin{aligned}\langle 0 | \bar{u}(0) \gamma_{\mu} \gamma_5 s(x) | K(p) \rangle &= i f_K p_{\mu} \int_0^1 du e^{-iup \cdot x} \left\{ \phi_K(u) + \frac{m_K^2 x^2}{16} A(u) \right\} \\ &\quad + f_K m_K^2 \frac{i x_{\mu}}{2 p \cdot x} \int_0^1 du e^{-iup \cdot x} B(u), \\ \langle 0 | \bar{u}(0) i \gamma_5 s(x) | K(p) \rangle &= \frac{f_K m_K^2}{m_s + m_u} \int_0^1 du e^{-iup \cdot x} \varphi_p(u), \\ \langle 0 | \bar{u}(0) \sigma_{\mu\nu} \gamma_5 s(x) | K(p) \rangle &= i(p_{\mu} x_{\nu} - p_{\nu} x_{\mu}) \frac{f_K m_K^2}{6(m_s + m_u)} \int_0^1 du e^{-iup \cdot x} \varphi_{\sigma}(u), \\ \langle 0 | \bar{u}(0) \sigma_{\alpha\beta} \gamma_5 g_s G_{\mu\nu}(vx) s(x) | K(p) \rangle &= f_{3K} \left\{ (p_{\mu} p_{\alpha} g_{\nu\beta}^{\perp} - p_{\nu} p_{\alpha} g_{\mu\beta}^{\perp}) - (p_{\mu} p_{\beta} g_{\nu\alpha}^{\perp} \right. \\ &\quad \left. - p_{\nu} p_{\beta} g_{\mu\alpha}^{\perp}) \right\} \int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i) e^{-ip \cdot x (\alpha_s + v \alpha_g)}, \\ \langle 0 | \bar{u}(0) \gamma_{\mu} \gamma_5 g_s G_{\alpha\beta}(vx) s(x) | K(p) \rangle &= f_K m_K^2 p_{\mu} \frac{p_{\alpha} x_{\beta} - p_{\beta} x_{\alpha}}{p \cdot x} \\ &\quad \int \mathcal{D}\alpha_i A_{\parallel}(\alpha_i) e^{-ip \cdot x (\alpha_s + v \alpha_g)} \\ &\quad + f_K m_K^2 (p_{\beta} g_{\alpha\mu} - p_{\alpha} g_{\beta\mu}) \\ &\quad \int \mathcal{D}\alpha_i A_{\perp}(\alpha_i) e^{-ip \cdot x (\alpha_s + v \alpha_g)}, \\ \langle 0 | \bar{u}(0) \gamma_{\mu} g_s \tilde{G}_{\alpha\beta}(vx) s(x) | K(p) \rangle &= f_K m_K^2 p_{\mu} \frac{p_{\alpha} x_{\beta} - p_{\beta} x_{\alpha}}{p \cdot x} \\ &\quad \int \mathcal{D}\alpha_i V_{\parallel}(\alpha_i) e^{-ip \cdot x (\alpha_s + v \alpha_g)} \\ &\quad + f_K m_K^2 (p_{\beta} g_{\alpha\mu} - p_{\alpha} g_{\beta\mu}) \\ &\quad \int \mathcal{D}\alpha_i V_{\perp}(\alpha_i) e^{-ip \cdot x (\alpha_s + v \alpha_g)},\end{aligned}\quad (22)$$

where the operator $\tilde{G}_{\alpha\beta}$ is the dual of the $G_{\alpha\beta}$, $\tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\mu\nu} G^{\mu\nu}$ and $\mathcal{D}\alpha_i$ is defined as $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. The light-cone distribution amplitudes are

parameterized as

$$\begin{aligned}
\phi_K(u, \mu) &= 6u(1-u) \left\{ 1 + a_1 C_1^{\frac{3}{2}}(2u-1) + a_2 C_2^{\frac{3}{2}}(2u-1) \right\}, \\
\varphi_p(u, \mu) &= 1 + \left\{ 30\eta_3 - \frac{5}{2}\rho^2 \right\} C_2^{\frac{1}{2}}(2u-1) \\
&\quad + \left\{ -3\eta_3\omega_3 - \frac{27}{20}\rho^2 - \frac{81}{10}\rho^2 a_2 \right\} C_4^{\frac{1}{2}}(2u-1), \\
\varphi_\sigma(u, \mu) &= 6u(1-u) \left\{ 1 + \left[5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho^2 - \frac{3}{5}\rho^2 a_2 \right] C_2^{\frac{3}{2}}(2u-1) \right\}, \\
\varphi_{3K}(\alpha_i, \mu) &= 360\alpha_u\alpha_s\alpha_g^2 \left\{ 1 + \lambda_3(\alpha_u - \alpha_s) + \omega_3 \frac{1}{2}(7\alpha_g - 3) \right\}, \\
V_{\parallel}(\alpha_i, \mu) &= 120\alpha_u\alpha_s\alpha_g (v_{00} + v_{10}(3\alpha_g - 1)), \\
A_{\parallel}(\alpha_i, \mu) &= 120\alpha_u\alpha_s\alpha_g a_{10}(\alpha_s - \alpha_u), \\
V_{\perp}(\alpha_i, \mu) &= -30\alpha_g^2 \left\{ h_{00}(1 - \alpha_g) + h_{01} [\alpha_g(1 - \alpha_g) - 6\alpha_u\alpha_s] \right. \\
&\quad \left. + h_{10} \left[\alpha_g(1 - \alpha_g) - \frac{3}{2}(\alpha_u^2 + \alpha_s^2) \right] \right\}, \\
A_{\perp}(\alpha_i, \mu) &= 30\alpha_g^2(\alpha_u - \alpha_s) \left\{ h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g - 3) \right\}, \\
A(u, \mu) &= 6u(1-u) \left\{ \frac{16}{15} + \frac{24}{35}a_2 + 20\eta_3 + \frac{20}{9}\eta_4 \right. \\
&\quad \left. + \left[-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3\omega_3 - \frac{10}{27}\eta_4 \right] C_2^{\frac{3}{2}}(2u-1) \right. \\
&\quad \left. + \left[-\frac{11}{210}a_2 - \frac{4}{135}\eta_3\omega_3 \right] C_4^{\frac{3}{2}}(2u-1) \right\} + \left\{ -\frac{18}{5}a_2 + 21\eta_4\omega_4 \right\} \\
&\quad \left\{ 2u^3(10 - 15u + 6u^2) \log u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \log \bar{u} \right. \\
&\quad \left. + u\bar{u}(2 + 13u\bar{u}) \right\}, \\
g_K(u, \mu) &= 1 + g_2 C_2^{\frac{1}{2}}(2u-1) + g_4 C_4^{\frac{1}{2}}(2u-1), \\
B(u, \mu) &= g_K(u, \mu) - \phi_K(u, \mu),
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
h_{00} &= v_{00} = -\frac{\eta_4}{3}, \\
a_{10} &= \frac{21}{8}\eta_4\omega_4 - \frac{9}{20}a_2, \\
v_{10} &= \frac{21}{8}\eta_4\omega_4, \\
h_{01} &= \frac{7}{4}\eta_4\omega_4 - \frac{3}{20}a_2, \\
h_{10} &= \frac{7}{2}\eta_4\omega_4 + \frac{3}{20}a_2, \\
g_2 &= 1 + \frac{18}{7}a_2 + 60\eta_3 + \frac{20}{3}\eta_4, \\
g_4 &= -\frac{9}{28}a_2 - 6\eta_3\omega_3,
\end{aligned} \tag{24}$$

here $C_2^{\frac{1}{2}}$, $C_4^{\frac{1}{2}}$ and $C_2^{\frac{3}{2}}$ are Gegenbauer polynomials, $\eta_3 = \frac{f_{3K}}{f_K} \frac{m_q+m_s}{m_K^2}$ and $\rho^2 = \frac{(m_u+m_s)^2}{m_K^2}$ [27, 28, 29, 30, 38, 39, 40, 41].

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